Unit 9 - Nature of Graphs

Unit 9 Day #1 - Symmetry and Coordinate Graphs

<u>Objective</u> - You will ① Determine if a graph is symmetrical by usinging algebra
② Classify functions as odd, even, or neither.

Symmetric about the y-axis

Rule $(-x, y)$

Ex: Is $y = x^2$ symmetric about the y-axis? Show algebmically and graphically.

<u>Algebraically</u>

$y = x^2$

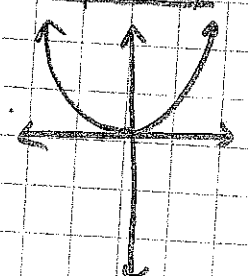
$y = (-x)^2$

$y = x^2$

<u>Same</u>

<u>Yes</u>

<u>Graphically</u>



<u>Yes</u>

Symmetric about the Origin

<u>Rule</u>: $(-x, -y)$

<u>Ex</u>: Is $y = x^3$ symmetric about the origin? Show algebraically and graphically.

<u>Algebraically</u>

$y = x^3$

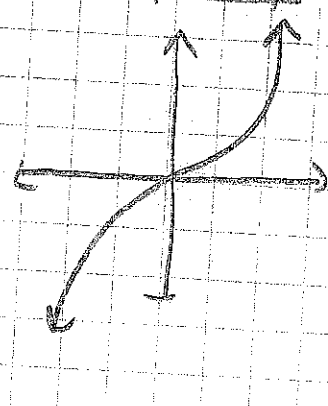
$-y = (-x)^3$

$-y = -x^3$

$y = x^3$

<u>Same</u> Yes

<u>Graphically</u>:



<u>Yes</u>

Symmetry with Polynomials

Even Functions: all even exponents
symmetric with the y-axis

Odd Functions: all odd exponents
symmetric with the origin

Determine if the following are symmetric with the y-axis, the origin or neither:

1) $y = 5x^{10} - 4x^4 + 3x^2$

Even Function / Symmetric to y-axis

2) $y = 6x^7 + 3x^3 - 4x$

Odd Function / Symmetric to origin

3) $y = -2x^5 + 3x^2 + 7x$

Different exponents / neither

Symmetry with Fractions

- Cannot use just the exponents to determine symmetry
- Must use the rules:

even function: y-axis $(-x, y)$
odd function: x-axis $(-x, -y)$

Determine if the following are even, odd or neither:

1) $y = \frac{1}{x^2 - 1}$

Even:

$y = \frac{1}{(-x)^2 - 1}$

$y = \frac{1}{x^2 - 1}$

Same!

Odd:

$y = \frac{1}{x^2 - 1}$

$-y = \frac{1}{(-x)^2 - 1}$

$-y = \frac{1}{x^2 - 1}$

NO

Even, y-axis

2) $y = \frac{1}{x - 1}$

Even:

$y = \frac{1}{-x - 1}$

$y = \frac{1}{-1(x+1)}$

NO

Odd:

$-1(-y) = \left(\frac{1}{-x - 1}\right) - 1$

$y = \frac{-1}{x + 1}$

NO

Neither

Homework:

1) $y = 5x^6 - 2x^4$

2) $f(x) = 2x^5 - 3x^3 + 2x$

3) $g(x) = 7x^5 - 4x^2$

4) $y = \dfrac{x^2 - 1}{x}$

5) $y^2 = \dfrac{4x^2}{9} - 4$

Unit 9 Day #2- Families of Graphs

Objective- You will
1. Identify transformations of simple graphs
2. Sketch graphs of related functions.

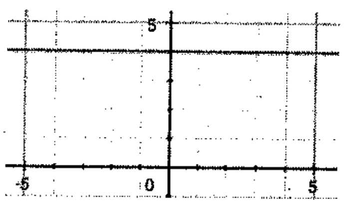
Families of Graphs - a group of graphs that display similar characteristics.

Parent Graphs - a basic graph that is transformed to make other graphs.

Examples of Parent Graphs

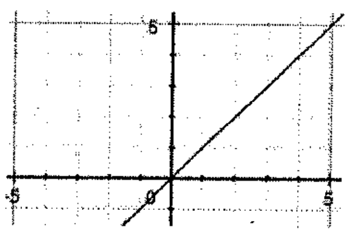
Constant Function

$y = \#$



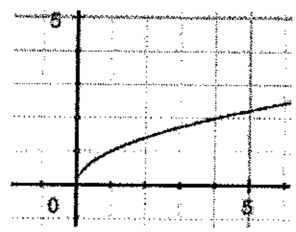
Identity Function

$y = x$



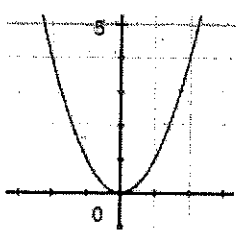
Square Root Function

$y = \sqrt{x}$

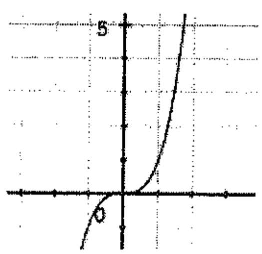


Polynomial Functions

$y = x^2$

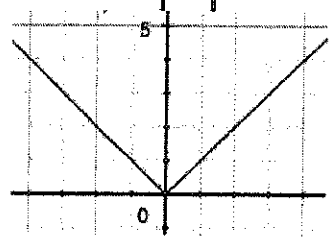


$y = x^3$



Absolute-Value Function

$y = |x|$



Transformations of Graphs

Reflect over the x: $y = -f(x)$

Reflect over the y: $y = f(-x)$

Translate Up: $y = f(x) + c$

Translate Down: $y = f(x) - c$

Translate Left: $y = f(x+c)$

Translate Right: $y = f(x-c)$

Examples: Determine the parent graph and then describe the transformations that take place to the parent graph to produce the following:

1) $y = (x+1)^3 - 2$

Parent Graph: $y = x^3$

Left 1
Down 2

2) $y = -(x-2)^2 + 5$

Parent Graph: $y = x^2$

- Right 2
- Reflect over x axis
- Up 5

3) $y = (-x)^3 + 2$

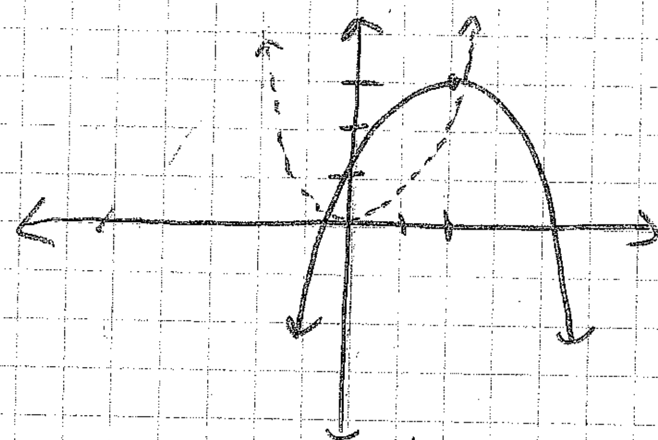
Parent Graph: $y = x^3$

- Reflect over y
- Up 2

4) Sketch $y = -(x-2)^2 + 3$

Parent graph: $y = x^2$

- Right 2
- Reflect over x-axis
- up 3



Homework: Identify the parent graph and describe the transformations that take to the parent graph.

1) $y = (x+1)^3 - 4$

2) $y = (-x)^3 - 7$

3) $y = -|x+2| + 5$

4) Sketch: $y = (x+4)^2 - 2$

Unit 9 Day #3 - Graphs of Non Linear Inequalities

<u>Objective</u> - You will ① Graph polynomial, absolute value, and radical inequalities in two variables

Graphing Non Linear Inequalities

1) Identify the parent graph

2) Describe the transformations that take place to the parent graph.

3) Sketch ... if $>$, dotted line & shade above
if $\geq$, solid line & shade above
if $<$, dotted line & shade below
if $\leq$, solid line & shade below

Example #1

<u>Given</u>: $y < -|x+2| + 3$

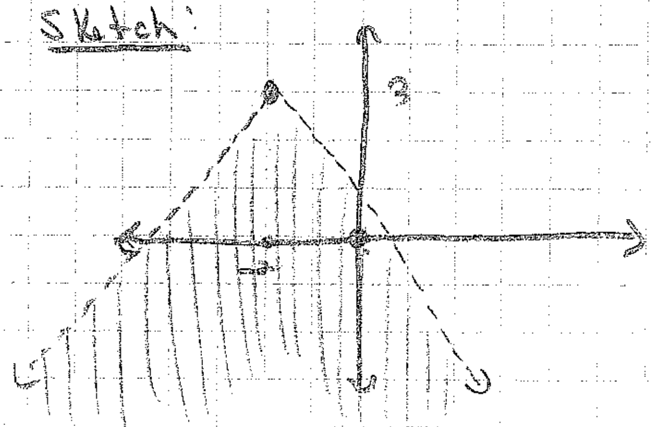
a) Identify the parent graph.

$y = |x|$

b) Describe the the transformations that take place to the parent graph to produce the given graph.

- Left 2
- Reflect over x-axis
- Up 3

c) <u>Sketch</u>:



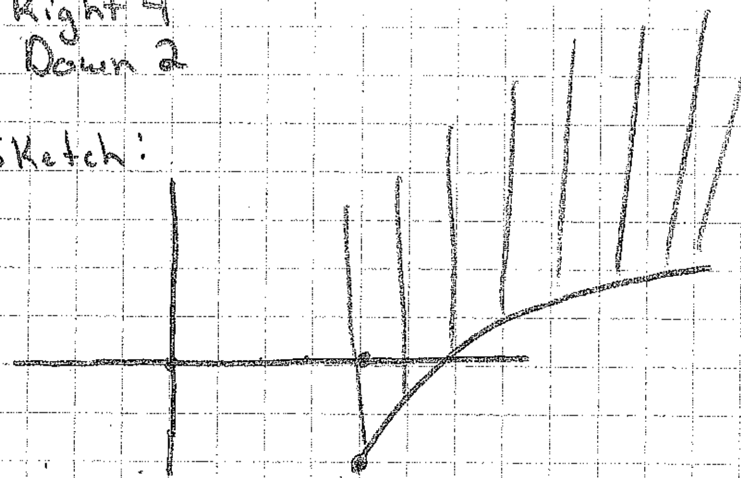
Example #2

<u>Given:</u> $y \geq \sqrt{x-4} - 2$

a) Parent Graph: $y = \sqrt{x}$

b) • Right 4
• Down 2

c) Sketch:



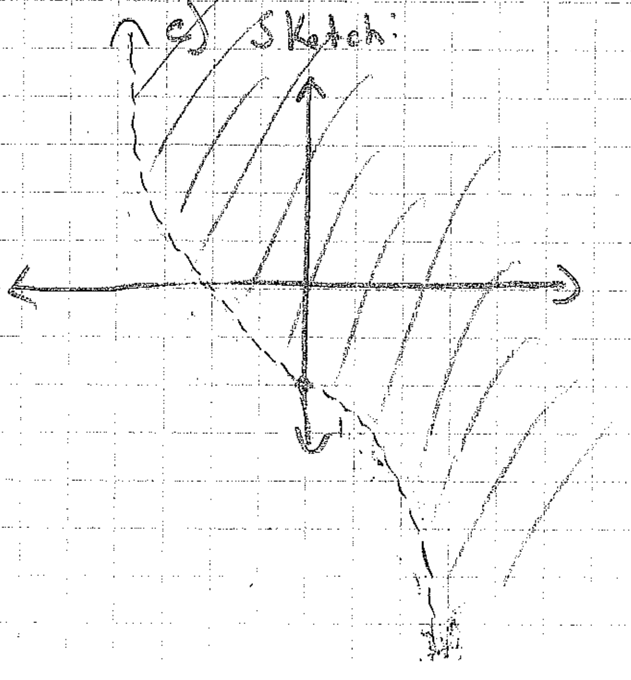
Example #3

<u>Given:</u> $y > (-x)^3 - 2$

a) <u>Parent Graph:</u> $y = x^3$

b) • Reflect over y-axis
• Down 2

c) Sketch:



Homework:

Identify the parent graph, describe the transformations that take to the parent graph and sketch.

1) $y < -(x+5)^2 - 3$

2) $y \geq |x-2| + 4$

3) $y < \sqrt{-x+2} - 5$

Unit 9 Day #4 - Inverse Functions

<u>Objective</u> - You will determine inverses of relations and functions.

Inverse Functions

Notation: $f^{-1}(x)$

Reflection over the line $y = x$.

To find the inverse, switch x and y, then solve for y.

To determine if $f^{-1}(x)$ is a function:

Check $f(x)$ with the horizontal line test.

OR

Check $f^{-1}(x)$ with the veritical line test.

Examples: Find the inverse and determine if the inverse is a function:

1) $f(x) = 3x - 5$

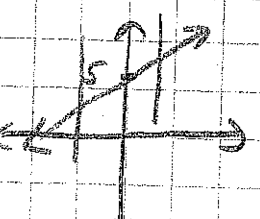
$f^{-1}(x) = 3y - 5$

$x = 3y - 5$

$\frac{x+5}{3} = \frac{3y}{3}$

$y = \frac{x+5}{3}$

$\boxed{f^{-1}(x) = \frac{x+5}{3}}$



Function Passes VLT

2) $f(x) = (x+3)^2 - 5$

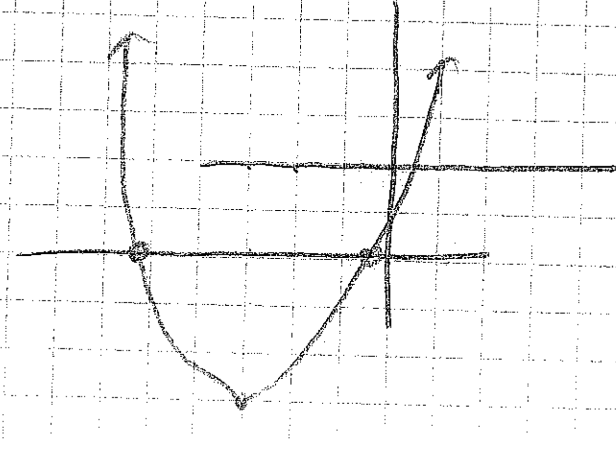
$x = (y+3)^2 - 5$

$\sqrt{x+5} = \sqrt{(y+3)^2}$

$\pm\sqrt{x+5} = y + 3$

$-3 \pm \sqrt{x+5} = y$

$\boxed{f^{-1}(x) = -3 \pm \sqrt{x+5}}$



Not a function since $f(x)$ did not pass the HLT

Verifying Inverses

To prove $f(x)$ and $f^{-1}(x)$ are inverses, show:

$$[f \circ f^{-1}](x) = [f^{-1} \circ f](x)$$

Ex: Find the inverse of $f(x) = 4x - 9$ and verify $f(x)$ and $f^{-1}(x)$ are inverses.

$$f(x) = 4x - 9$$

$$x = 4y - 9$$

$$x + 9 = 4y$$

$$y = \frac{x+9}{4}$$

$$\boxed{f^{-1}(x) = \frac{x+9}{4}}$$

$$[f \circ f^{-1}](x) = f\left(\frac{x+9}{4}\right)$$

$$= \cancel{4}\left(\frac{x+9}{\cancel{4}}\right) - 9$$

$$= x + 9 - 9$$

$$\boxed{= x}$$

$$[f^{-1} \circ f](x) = f(4x-9)$$

$$= \frac{(4x-9)+9}{4}$$

$$= \frac{4x}{4}$$

$$\boxed{= x}$$

$\therefore$ Inverses

Homework: Find the inverse and determine if the inverse is a function.

1) $f(x) = 2x + 7$

2) $f(x) = -\frac{1}{x^2}$

3) $f(x) = (x-3)^2 + 7$

Unit 9 Day #5 - Piecewise Functions

<u>Objective</u> - You will sketch a function that contains two or more pieces of diff. functions at certain intervals.

<u>Piecewise Functions</u>

- Several graphs drawn on the same grid over different intervals.
- Use table of values to get the shape of each graph.
- When the interval is $<$ or $>$, start with that number and use an open circle.
- When the interval is $\leq$ or $\geq$, start with that number and use a closed circle.

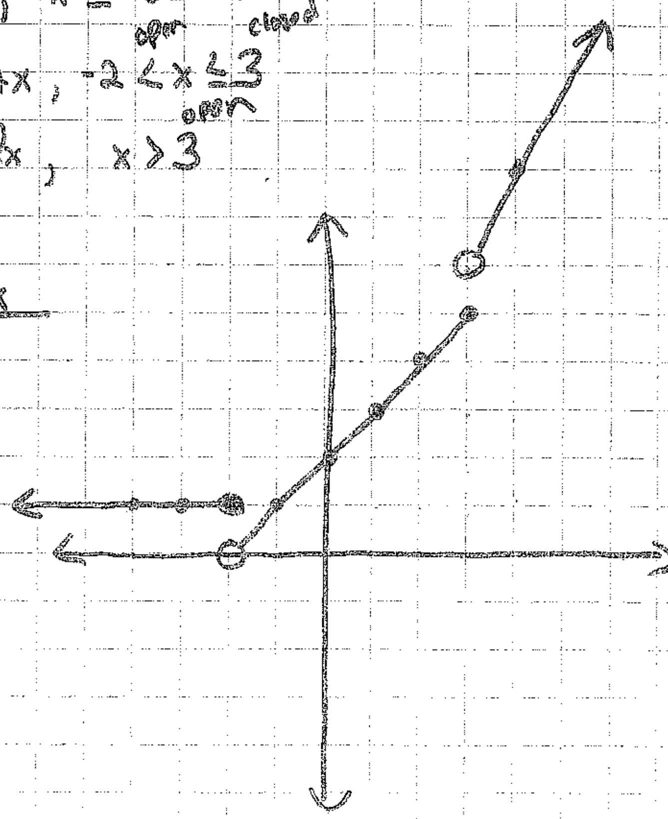
Example #1

sketch: $f(x) = \begin{cases} 1, & x \leq -2 \\ 2+x, & -2 < x \leq 3 \\ 2x, & x > 3 \end{cases}$

closed
open closed
open

x	1
-4	1
-3	1
-2	1

x	2x
3	6
4	8
5	10



x	2+x
-2	0
-1	1
0	2
1	3
2	4
3	5

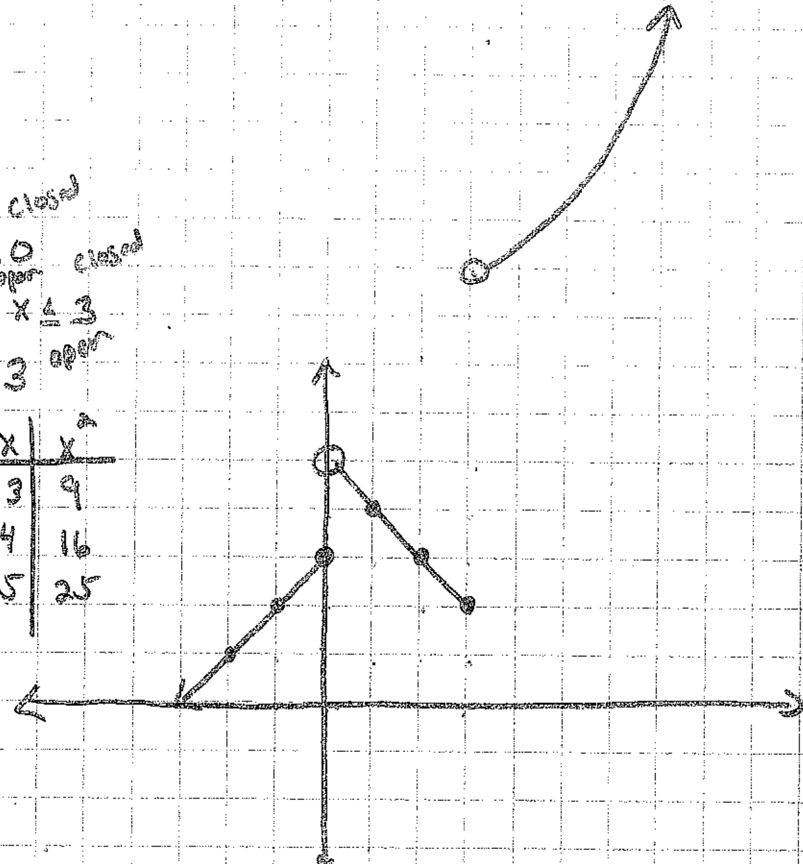
Example #2:

Sketch: $f(x) = \begin{cases} x+3, & x \le 0 \ \text{closed} \\ 5-x, & 0 < x \le 3 \ \text{open, closed} \\ x^2; & x > 3 \ \text{open} \end{cases}$

x	x+3
-2	1
-1	2
0	3

x	5-x
0	5
1	4
2	3
3	2

x	x^2
3	9
4	16
5	25



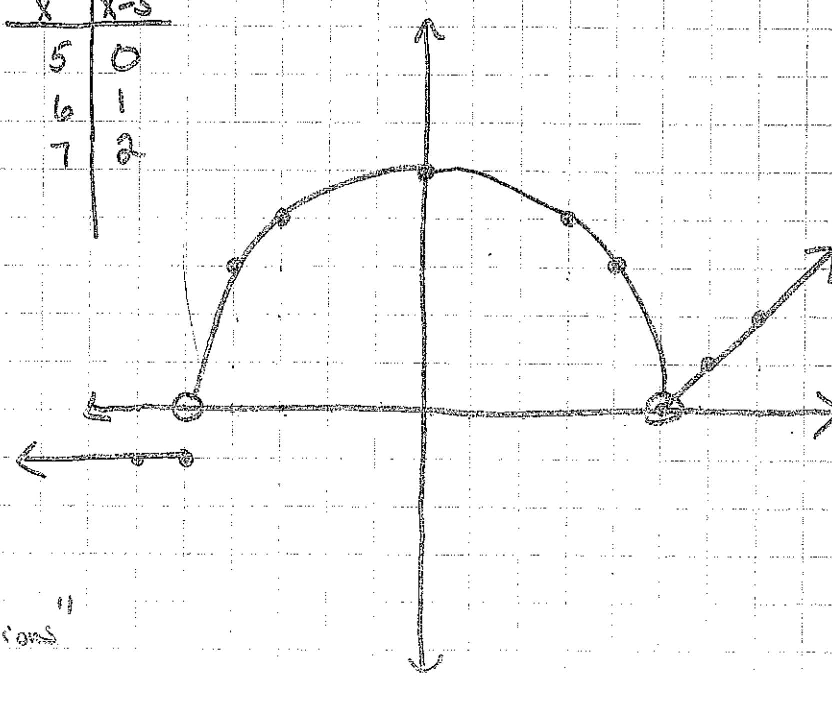
Example #3

Sketch: $f(x) = \begin{cases} -1, & x \le -5 \ \text{closed} \\ \sqrt{25-x^2}, & -5 < x < 5 \ \text{open open} \\ x-5, & x \ge 5 \ \text{closed} \end{cases}$

x	-1
-6	-1
-5	-1

x	$\sqrt{25-x^2}$
-5	0
-4	3
-3	4
-2	
-1	
0	5
1	
2	
3	4
4	3
5	0

x	x-5
5	0
6	1
7	2



Homework - WS "Piecewise Functions"

Unit 9 Day 6 - Continuity

Objective - You will Determine: ① If a function is continuous or discontinuous
② The end behavior of a function

Continuity

Continuous Function - when a function can be drawn in one motion (without lifting your pencil)

Discontinuous Function - when a function cannot be drawn in a single motion.

Types of Discontinuity

Infinite - $|f(x)|$ becomes greater and greater as the graph approaches a #. (this occurs where the graph is undefined)

ex: $f(x) = \frac{1}{x^2}$

Jump - the graph stops a given value and then starts again in a different location.

ex: piecewise functions

Point - when there is a value where the graph is undefined (open circle) but the pieces of the graph match up

ex: $f(x) = \frac{x^2-4}{x-2}$ Hole @ common factor $= 0$
$x-2=0$
Hole @ $x=2$

End Behavior

Describes what happens to the graph as you go to the left $(x \to -\infty)$ as you go to the right $(x \to +\infty)$.

Does the graph go up $(y \to +\infty)$, does it go down $(y \to -\infty)$ or does it 'level off' $(y \to \#)$?

Focus on the term with the highest powered exponent

Examples: Describe the end behavior of the following:

1) $f(x) = \boxed{x^{10}} + x^9 + 5x^8$

As $x \to +\infty$, $f(x) \to +\infty$

As $x \to -\infty$, $f(x) \to +\infty$

2) $f(x) = \boxed{5x^7} + 3x^4 - 2x^2$

As $x \to +\infty$, $f(x) \to +\infty$

As $x \to -\infty$, $f(x) \to -\infty$

3) $f(x) = \boxed{-4x^7} + 5x^6 - 3x^2$

As $x \to +\infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to +\infty$

4) $f(x) = \dfrac{-1}{x^2+2}$

As $x \to +\infty$, $f(x) = 0$

As $x \to -\infty$, $f(x) = 0$

Homework: Describe the end behavior

1) $f(x) = x^3 + 2x^2 + x - 1$

2) $f(x) = 8 - x^3 - 2x^4$

3) $f(x) = |(x-2)^2 - 1|$

4) $f(x) = \dfrac{1}{x^2}$

Unit 9 Day #7 - Limits

<u>Objective</u>: You will find the limit graphically.

<u>Evaluating Limits Graphically</u>

Notation: $\lim\limits_{x \to c} f(x)$

"the limit of f(x) as x approaches c."

<u>Meaning</u>: What # is the y-value (f(x)) approaching as x approaches a # (c-value)?

<u>Right-Hand and Left-Hand Limits</u>

<u>Right-Hand Limit</u>: x approaches c from the right side

$\lim\limits_{x \to c^+} f(x)$

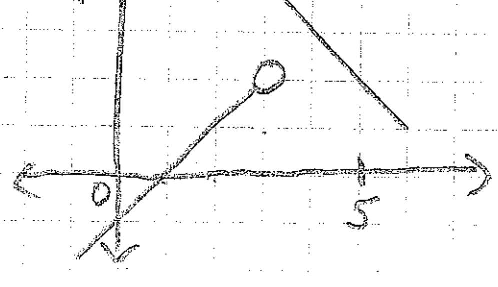
<u>Left-Hand Limit</u>: x approaches c from the left side

$\lim\limits_{x \to c^-} f(x)$

<u>Two-Sided Limit</u>: exists only when the left and right side limits are the same.

$\lim\limits_{x \to c} f(x)$

Example #1: Evaluate the following limits



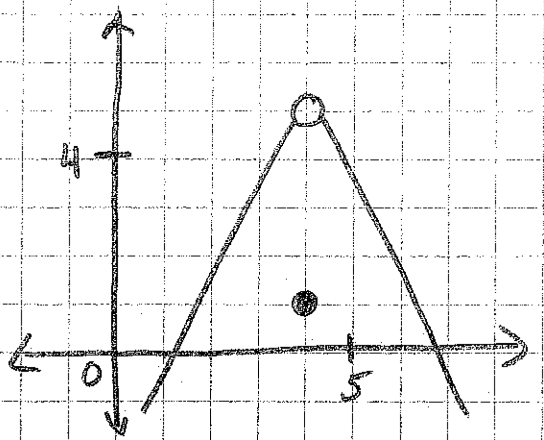
a) $\lim\limits_{x \to 3^-} f(x) = 2$

b) $\lim\limits_{x \to c^+} f(x) = 4$

c) $\lim\limits_{x \to 3} f(x) =$

d) $f(3) = 4$

Example #2: Evaluate the following limits:



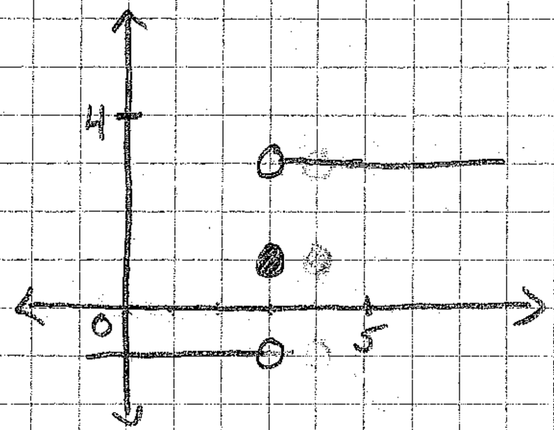
a) $\lim_{x \to 4^-} f(x) = 5$

b) $\lim_{x \to 4^+} f(x) = 5$

c) $\lim_{x \to 4} f(x) = 5$

d) $f(4) = 1$

Example #3 - Evaluate the following Limits



a) $\lim_{x \to 3^-} f(x) = -1$

b) $\lim_{x \to 3^+} f(x) = 3$

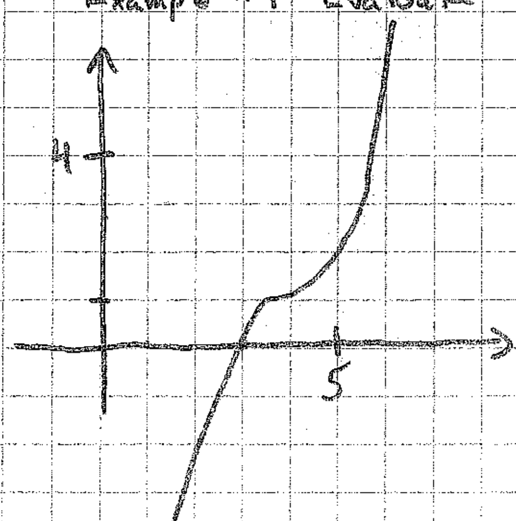
c) $\lim_{x \to 3} f(x) = \text{DNE}$

d) $f(3) = 1$

e) $\lim_{x \to -\infty} f(x) = -1$

f) $\lim_{x \to +\infty} f(x) = 3$

Example #4 - Evaluate the following Limits



a) $\lim_{x \to 4^-} g(x) = 1$

b) $\lim_{x \to 4^+} g(x) = 1$

c) $\lim_{x \to 4} g(x) = 1$

d) $\lim_{x \to -\infty} g(x) = -\infty$

e) $\lim_{x \to +\infty} g(x) = +\infty$

HW WS
2-10 even

Unit 9 Day #8 - Computing Limits as $x \to \#$.

Objective - You will determine the limit as $x \to \#$.

Computing Limits as $x \to \#$

1) Substitute the # into the function

2) If you get:

- an answer - that's it! You're done!
- $\frac{0}{0}$ - factor, reduce & resubstitute.
- $\frac{\#}{0}$ - the answer is DNE and is either $+\infty$ or $-\infty$ (use a sign analysis to determine which)

Examples

1) $\lim_{x \to 3} \frac{x^2+2x-5}{x+2}$

$= \frac{(3)^2+2(3)-5}{3+2}$

$= \frac{9+6-5}{5} = \frac{10}{5} = \boxed{2}$

2) $\lim_{x \to 5} \frac{x^2-25}{x-5}$

$= \frac{5^2-25}{5-5} = \frac{0}{0}$

$\frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}}$

$x+5 = 5+5 = \boxed{10}$

★ 3) $\lim_{x \to 4} \frac{4-x}{2-\sqrt{x}}$

$= \frac{4-4}{2-\sqrt{4}} = \frac{0}{0}$

$\frac{4-x}{2-\sqrt{x}} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} = \frac{8+4\sqrt{x}-2x-x\sqrt{x}}{4-x}$

$= \frac{4(2+\sqrt{x})-x(2+\sqrt{x})}{4-x}$

$\frac{\cancel{(4-x)}(2+\sqrt{x})}{\cancel{4-x}}$

$2+\sqrt{x}$

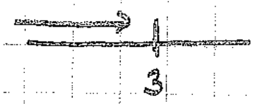
$= 2+\sqrt{4}$

$= 2+2$

$\boxed{= 4}$

Evaluate:

4) $\lim_{x \to 3^-} \frac{x}{x-3} = \infty$



$\frac{3}{3-3} = \frac{3}{0}$ $\boxed{DNE}$

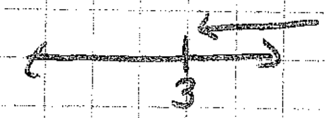
↑

bad Ans.

$\frac{2.9999999}{2.9999-3} = \frac{+}{-} = - \quad \therefore = \boxed{-\infty}$

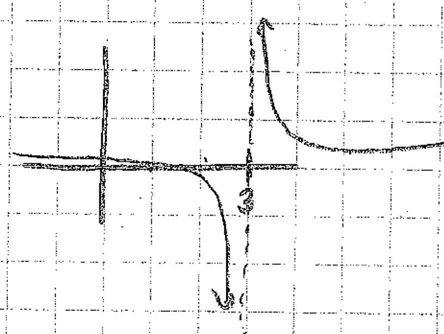
5) $\lim_{x \to 3^+} \frac{x}{x-3} = \infty$

$\frac{3}{3-3} = \frac{3}{0}$ $\boxed{DNE}$



$\frac{3.0000001}{3.0000001-3} = \frac{+}{+} = + \quad \therefore = \boxed{+\infty}$

6) $\lim_{x \to 3} \frac{x}{x-3} = \boxed{DNE}$



HW: WS #6-8, 11, 12, 18-20, 29

Unit 9 Day #9 ⇒ Computing Limits as $x \Rightarrow \pm\infty$

Objective - You will find the limit as $x \Rightarrow \infty$

Overall Question: Which is getting larger, faster? The numerator? The denominator?

When taking the limit as $x \to +\infty$ or $x \to -\infty$, look for the term with the highest powered exponent.

If the highest powered exponent is in:

Denominator: answer is 0.

Numerator: answer is $x \to +\infty$ or $x \to -\infty$ (sign analysis)

Both: answer is the coefficient of these terms.

Examples: Evaluate:

1) $\lim_{x \to +\infty} \dfrac{x+3}{\boxed{4x^2}} = \boxed{0}$

2) $\lim_{x \to -\infty} \dfrac{5x \boxed{-3x^4}}{3 + \boxed{6x^3}} = \dfrac{-3(-)^4}{6(-)^3} = \dfrac{-}{-} = +$ $\boxed{= +\infty}$

$-\infty$ or $+\infty$

3) $\lim_{x \to -\infty} \dfrac{\boxed{14x^3} + 5x}{4 - 7x^2}$

$= \dfrac{14}{-7} = \boxed{-2}$

4) $\lim_{x \to \infty} \dfrac{-3x^2 + 2x}{1} = \boxed{-\infty}$

$-\infty$ or $+\infty$

$-3(+)^2 = -$

5) $\lim_{x \to -\infty} \dfrac{3}{x+4} = \boxed{0}$

6) $\lim_{x \to -\infty} \dfrac{\sqrt{2x^4 - 5x^2}}{3x^2}$

$= \dfrac{\sqrt{2}\sqrt{x^4} - \sqrt{5x^2}}{3x^2}$

$= \dfrac{\boxed{\sqrt{2}\,x^2} - \sqrt{5}x}{\boxed{3}x^2} = \boxed{\dfrac{\sqrt{2}}{3}}$

7) $\lim_{x \to \infty} \dfrac{\sqrt{3x^8 - 2x + 1}}{-5x^4 - 9} = \dfrac{\sqrt{3}\,x^4}{-5x^4}$

$\boxed{= \dfrac{\sqrt{3}}{-5}}$

HW: WS # 7, 12, 15, 16, 20, 24, 26